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Development & Incorporation of a New Smoke Module for the Vector-In-Commander Model

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Dennis L. Bechtloffl
Daniel W. Tulloh

Final Report

DEPARTMENT OF THE ARMY

August 1990

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Department of the Army
TRADOC Analysis Command
White Sands Missile Range, NM 88002-5502

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13. ABSTRACT *(Maximum 200 words)*

The difficulty with factoring smoke/dust into combat power is that these obscurants are, in general, not lethal. In an effort to determine the effects of these obscurants, the US Army initiated the Comprehensive Smoke Study (CSS). At the corps level, the proponents of the CSS intended to represent the many different aspects of smoke usage, to include logistics, the actual deployment of the obscurant, and an approximation of its effects. The Army's Corps level combat model, Vector-In-Command (VIC), did not have a smoke module capable of representing these different aspects of smoke usage. To remedy this deficiency, the Corps Support Division of the Combat Simulation Directorate at TRAC-WSMR was tasked to develop a smoke module capable of meeting these requirements. A product of the Atmospheric Sciences Laboratory called the Combined Obscuration Model for Battlefield-Induced Contaminants (COMBIC) portrays the various delivery systems and obscurant types the CSS wished to employ. However COMBIC is a high-resolution model, in contrast with the VIC model which simulates the battlefield at an aggregate level. These differences forced the VIC modelling team to make modifications to COMBIC before it was feasible to use it in the new smoke module. VIC's implementation of COMBIC now allows the model to: portray wide range of obscurants and delivery systems, dynamically portray smoke/dust screens, simplify the calculation of degradation factors, remove screens too thin to degrade sensors, impact movement, sensor performance and direct-fire attrition in the presence of obscurants.

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Table of Contents

Development & Incorporation of a New Smoke Module for the VECTOR-In-Commander Combat Model

Introduction

Commanders of troops in battle have always been plagued by one overriding concern: How to gain accurate information on the strength and movements of the enemy without their opponents learning the same information about themselves. Those who were able to achieve this goal often enjoyed a tremendous advantage over their rivals. Commanders of today's Army are well aware of the tactical advantages that can be gained from the acquisition and denial of battlefield intelligence. Due to the long range and accuracy of modern direct-fire weapons, the battlefield commander must use sensors to enhance his intelligence-gathering capability. These sensors, however, can be severely affected by the presence of obscurants. In order to evaluate the impact of these obscurants on operational effectiveness, the US Army initiated the Comprehensive Smoke Study (CSS).

Background & COMBIC Capabilities

The CSS analysis was to be performed in two phases. Phase 1 was designed to analyze operational effectiveness at the battalion level while Phase 2 would investigate obscurant operations at the corps level. The CSS proponents chose the Vector-In-Commander (VIC) combat model as the tool which they would use to complete the latter portion of their study. VIC is a two-sided, event sequenced, deterministic simulation of combat representing both land and air forces at the corps level. At the beginning of Phase 2 of the CSS the proponents identified a list of critical requirements for the conduct of their analysis. These requirements were:

- The need to represent the employment of many different kinds of obscurants,
- The need to represent several delivery systems, and
- The need to represent corps-level smoke operations and the effects of smoke usage.

The existent representation of obscurants and their effects already in the VIC model was unable to adequately meet these demands. A search of available smoke models was conducted to find a representation of obscurant effects capable of meeting these needs and be compatible with the VIC model. No such models were found. However, the model team did find a high-resolution smoke model that represented many different kinds of obscurants and delivery systems. This model is called the Combined Obscuration Model for Battlefield Induced Contaminants (COMBIC). A product of the Atmospheric Sciences Laboratory, located at the White Sands Missile Range (WSMR), New Mexico COMBIC is capable of representing a multitude of delivery systems and obscurant types. The VIC modelers therefore chose COMBIC to be the basis for the development of a new smoke module.

Modelling Methodology

Aggregate-level models such as VIC do not lend themselves well to the direct inclusion of high-resolution models due to the large number of computations and the amount of storage required. Extreme care is required in the aggregation of these types of models in order to preserve their integrity. Analysis of COMBIC's methodology and output proved that many concepts could survive the aggregation process and be adapted for use in the VIC model. Once this was ascertained, development began on an aggregate version of COMBIC for inclusion into the VIC model.

Development

Analysis of COMBIC

The COMBIC model is divided into two portions - the initialization (Phase 1) and the scenario (Phase 2). Phase 1 develops and stores an obscurant downwind profile by utilizing a source identifier and various meteorological data. This downwind profile is a distance-stepped array that contains information about the leading edge of the cloud. This information includes the time-of-arrival, the crosswind spread, the height attained, and the total amount of obscurant mass produced by the source at various downwind distances. Phase 2 locates an observer-target pair and the source in a terrain area and then performs the sensor electro-magnetic transmission (EMT) degradation calculations. For full details on the representation of a smoke/dust cloud in COMBIC, see Chapter 1 of the Electro-Optical Systems Atmospheric Effects Library (EOSAEL) 82 Volume III, *Transmission Through Battlefield Aerosols*, US Army Electronics Research and Development Command Atmospheric Sciences Laboratory, November, 1982.

In other words, the COMBIC representation of smoke can be likened to a triangular wedge as shown in Figure 1. At the Corps level, there are so many active smoke/dust sources that using this representation in the VIC model would be unwieldy in terms of its storage and computational requirements. Therefore it was necessary to find some way to modify this representation to make it usable at the aggregate level.

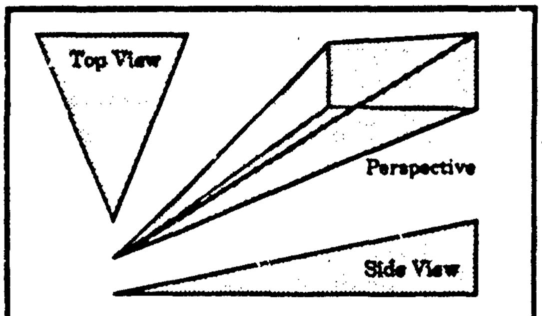

Figure 1: COMBIC representation of a single source obscurant cloud

Conversion of COMBIC to an Aggregate Form

Combat units do not request single smoke/dust sources, but rather smoke/dust SCREENS. The size of these screens can be large - some of the screens called for may contain up to 500 sources and be over four kilometers long. Since each one of these sources produces a triangular-shaped cloud, arranging these sources in a line allows the shape of the whole screen to be likened to a trapezoidal wedge (See Figure 2). This description gives us an adequate representation of a smoke screen without excessive deviation from the reality model. Through the use of the COMBIC-generated single source downwind profiles, the dimensions of the screen can be determined from the sources at either end of the smoke line. The amount of obscurant mass in the screen is found by using the cumulative amount of obscurant produced by the single source and multiplying that value by the number of sources comprising the screen. Thus these source profiles can be used to let the screen representation grow as time passes and drift downwind after the sources stop injecting obscurant into the screen. This information is then used to approximate the correct sensor EMT degradation.

The screen representation for the VIC model required another modification before its use became practical. When the VIC model needs a degradation calculation to be performed, it uses a single line to represent the EMT calculation for a group of weapons that are dispersed along a line. This level of resolution presents a problem when this Line-Of-Sight (LOS) projection happens to intersect a smoke/dust screen. When a degradation calculation for a particular group of

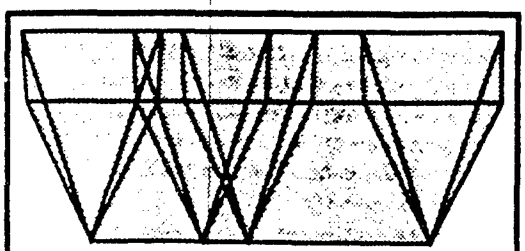

Figure 2: VIC screen representation showing individual clouds

weapons is performed, the LOS could pass between the individual sources and suffer no loss in EMT intensity or it might pass directly over a source and result in a tremendous loss in intensity. Yet this number is applied to all firing weapons in that group. A better representation of calculating an EMT from the observer to the target was required in order to adequately capture the effects of obscurants on the direct fire battle in the VIC model.

Since a single LOS position calculation is used to represent a degradation for a number of weapons, the problem was how to make the EMT degradation along this LOS projection a reasonable approximation to the real-life expectation. The solution was to make the screen have a constant density of obscurant throughout its volume. Thus there are no holes, and there are no very dense zones either. An average density is easily computed by taking the ratio ***M/V***, where ***M*** is the total mass of obscurant in the screen and ***V*** is the screen volume. (A derivation of the screen volume can be found in appendix A.) Note that most smoke screens are still capable of effectively interfering with acquisition after their sources stop obscurant production for some length of time. This means that the calculation of the screen volume requires a computation for both the volume at the leading edge and at the trailing edge. This is because these edges are related to time, the leading edge to the ignition time of the source and the trailing edge to the time of source 'burn out.' Once the sources burn out, the smoke begins to drift away from the line of sources which would result in a overestimate of the screen volume. To arrive at a better estimate of the true volume, the volume calculation treats the screen as a solid entity from the line of sources to the edge in question. Calculating a volume from the smoke line to the leading edge and another value for the smoke line to the trailing edge and then finding the difference yields a better answer.

$$M = M(t) \times n \tag{1}$$

where

$M(t)$ is the amount of mass produced by a single source in t seconds, and

n is the number of sources in the screen.

$$V_{ld} = \frac{X_{ld} \times Z_{ld} \times \left(S + 2 \times B_{ld} \right)}{6.0} \tag{2}$$

where

V_{ld} is the screen volume from the ignition line to the screen's leading edge

X_{ld} is the distance from the source to the leading edge

Z_{ld} is the screen's height at the leading edge

S is the length of the base line

B_{ld} is the length of the screen at the leading edge

$$V_{tr} = \frac{X_{tr} \times Z_{tr} \times \left(S + 2 \times B_{tr} \right)}{6.0} \tag{3}$$

where

V_{tr} is the screen volume from the ignition site to the screen's trailing edge

X_{tr} is the distance from the source to the trailing edge

Z_{tr} is the screen's height at the trailing edge

S is the length of the base line

B_{tr} is the length of the screen at the trailing edge

$$D = \frac{M}{V_{ld} - V_{tr}} \tag{4}$$

where

D is the average density of the screen

Note that X_{tr} could equal zero if the source is still active (i.e. still producing obscurant), thus forcing V_{tr} to equal zero. There are some modifications to these equations if the screen line placement is nearly parallel to the wind direction, but these do not adversely impact the outlined methodology.

The methodology used to calculate the amount of degradation in transmission along an LOS projection in the VIC model is essentially the same method used by COMBIC. This method involves the use of a relationship known as the Beer-Lambert Law. This formula states that:

$$T = e^{-\alpha C_l} \qquad (5)$$

where

- T is the percent transmission remaining,
- α is the extinction coefficient, and
- C_l is the concentration length.

The extinction coefficient is a scalar value which provides a measure of a particular obscurant's ability to interfere with sensor performance on a given wavelength. The concentration length is a path integral which yields the amount of obscurant along the LOS projection. This LOS projection and the obscurant delivery method are factors in the evaluation of this integral - a short path through dense obscurant could yield the same C_l value as a long path through a thinner one.

The new representation of smoke/dust screen in the VIC model allows us to greatly simplify the computation of the degradation along an LOS projection. As previously stated, an obscurant screen is represented by a trapezoidal wedge of uniform density. Because the amount of obscurant mass per integration step is the same, the computation of the path integral can be reduced to a simple length times density function. This means that once the amount of the LOS projection encountering obscurant is determined, all that is required is to multiply this length by the screen density to obtain a C_l value.

$$C_l = l \times d \qquad (6)$$

where

- C_l is the concentration length,
- l is the path-length through the obscurant
- d is the average density of smoke/dust screen

A new methodology was also provided for the removal of screens that are too thin to degrade sensors. When a smoke screen is initially created it is placed into a list of active smoke screens. Every two minutes the VIC model calls a routine that updates the attributes of the screen. These attributes include such things as size, density, and the time since source ignition. At the time of the update the VIC model also performs tests to determine whether or not the screen should remain in the active list. A screen is removed if it meets one of the following conditions:

- A test calculation shows that the C_l value is below 0.01, or
- The User-defined cutoff value beyond source 'burn-out' is exceeded.

The representation of a dust screen formed by an artillery barrage is similar to that for smoke, but there are some notable differences. Naturally the HE rounds from an artillery volley do not all land at once or on a straight line and representing the screen in this way would stretch its credibility. COMBIC provides the ability to combine the dust produced from each of these rounds using a barrage option. With this option we only need to specify

1) the production rate in rounds/hectare/minute,
2) the area of impact in hectares,
3) the barrage duration in minutes, and
4) the soil type where the rounds impact.

These values are calculated in the new VIC model methodology with the exception of the soil type. The soil type is presently set to a value indicating a damp cohesive soil. A future enhancement could provide an input soil type to be entered with the other terrain parameters in the VIC model. The barrage option collapses all delivered rounds into a single source with an appropriate downwind history profile. Since a screen is represented as a trapezoidal wedge, this presents a problem as a baseline can not be determined from a single source location. COMBIC represents obscurant clouds as Gaussian entities described by obscurant dispersion sigmas in the component coordinates. These sigmas are the standard deviations of the Gaussian distributions. Multiplying these sigmas by 2.15 encloses 97 percent of the obscurant produced by the source. This is incorporated in VIC by multiplying the radius of the targeted weapons by 2.15 and then using that value for the source baseline.

Integration Into the VIC Model

Partial Screening

The methodology used for Front-Line attrition places the weapons of a particular type within the affected unit quadrant on a line at some user-specified distance from the center of the unit. Thus there will be two lines of opposing weapon systems trying to attrite each other. To determine the amount of firing weapons able to see the target weapons, a number of LOS projections are drawn. A single LOS projection taken from the center of the firing unit to the center of the target unit is used to determine the Probability-of-LOS and detection rates for the firing weapons. The proponents of the CSS were concerned that the use of this methodology in determining obscuration effects of smoke/dust screens would not adequately represent the effects of partial screening. If the LOS projection did not intersect any screens, then it was as if the entire line of weapons could see clearly. To alleviate these concerns, the VIC model was modified to use three LOS's to determine the degradation due to the presence of obscurants. The line of firing weapons is divided into thirds and a LOS projection is made from the center of each third to the center of the corresponding third in the line of target systems, computing a degradation for each LOS path. (See Figure 3.) By averaging the degradations for these three LOS projections, a more reasonable degradation calculation is made for the unit. This average is then applied to the attrition coefficient for the unit as a linear multiplier to the number of firing weapons.

Artillery Enhancements

The implementation of the new smoke module necessitated a number of enhancements to be made to the VIC artillery methodology. The most difficult of these enhancements was the addition of a blinding-smoke mission for the red side. These types of missions consist of a mixture of smoke and high explosives (HE) rounds providing a combination of lethality and obscuration effects. The VIC model, originally would not allow an artillery battery to fire a combination of munition types in a single mission, nor was it able to schedule two separate missions and provide any of coordination between them. The new smoke representation in the VIC model is able to perform a blinding smoke mission, which includes both HE and smoke rounds, by recoding parts of the artillery module so that the blinding smoke mission is composed of two munition types. The only restriction is that the mission must deliver both the HE and the smoke rounds by the same artillery fire group.

Different Attrition Templates

A necessary enhancement to the VIC artillery methodology was the placement of the dust screen generated by the arrival of an HE barrage. This enhancement was necessary due to the two unit 'templates' that the VIC attrition methodology uses. These two templates are: 1) an artillery template which treats the unit as a homogeneous disk and 2) a direct-fire template which places a units weapons within the engaged quadrant at a user-defined distance from the unit center. The artillery targets the center of the target unit so that is where the dust screen was initially placed. Analysis revealed that these dust screens were not affecting the direct-fire attrition because of the template difference. In order to achieve a representation of dust effects on the direct fire battle the dust screen was placed at the front of the targeted unit. This was where most of the units weapons

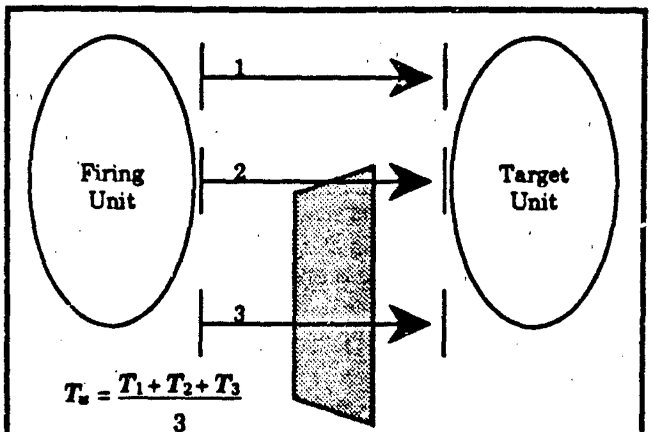

Figure 3: Three LOS's to represent effects of partial screening

were deployed and where it was most likely that a forward-observer would target the artillery mission.

Effects of Short-Lived Screens

Some of the smoke screens that are created have 'life spans' that are shorter than the cycle time for the Front-Line Attrition (FL) module. The problem was how to be confident that these smoke screens would affect the battle no matter when they would appear. The answer turned out to involve the affected unit set. Basically, the affected unit set is a catch-all set which forces various units to access the decision tables again because an event occurred that might affect them. They may wind up in combat again, decide to withdraw, request reinforcements, and so forth. The solution was to draw a rectangular box around the screen and then load any units whose centers fell within the box into the affected unit set. The dimensions of this box are calculated by finding the distance from the center to the screens' edge and adding a user-specified distance. This distance is then multiplied by two to give a dimension of the box.

Effects of Smoke/Dust

The appearance of smoke/dust can have an adverse effect on unit movement, acquisition, and attrition. These effects are now represented in the following manner:

Movement

A unit that encounters smoke has its movement slowed until the smoke/dust screen clears or the unit moves out of the screen. This fact is taken into account by projecting a LOS out to a user-defined distance in front of the unit in the direction of movement. A degradation calculation along this projection is performed and this value is applied as a linear multiplier to the units speed. This procedure has the effect of decreasing a units speed to a user-defined level.

$$New\ Speed = Old\ Speed \times Degradation \quad (7)$$

Acquisition

There are two types of acquisition that occur in the VIC model; one is performed by the sensing units and the other is performed by the maneuver units. The sensing units are those units whose observations go through a filtering process for inclusion into the intelligence data base. A sensing unit is composed of a number of individual sensors whose probability of detection is determined by:

- Percent of detected weapon mass
- The 'intelligence emissions' of the target
- The probability of LOS to the target
- and a smoke factor

The smoke factor is applied as a linear multiple to the individual sensors probability of detection. The fraction of the unit actually detected is then determined by the following formula;

$$FRAC_d = \left(1.0 - \left[1.0 - P_{det}\right]^n\right) \times FRAC_i \quad (8)$$

where
$FRAC_d$ is the fraction of unit detected
P_{det} is the sensor probability of detection of the target
n is the number of sensors in the sensing unit
$FRAC_i$ is the fraction of unit inside the footprint

The maneuver units use an acquisition factor to determine their attrition coefficients for use in the direct fire attrition methodology. However, a degradation factor cannot be applied to both the acquisition rate and the attrition rate since this would over-assess the effect of obscurants.

Attrition

Those units which enter combat use the Bonder-Farrell Differential Equation Methodology to determine casualties and enemy losses. Using the method of the three LOS projections outlined earlier, the calculated degradation factor is applied to the attrition coefficient as a linear multiplier. For more details concerning the Bonder-Farrell Differential Equation Methodology of attrition, refer to the FL module documentation in the VIC User Guide.

Comparison with COMBIC

Scenario Description

After the screen representation was put into computer code and the verification tests were complete, a number of validation test runs were made to compare the new representation with the results of the COMBIC model. Twenty-one obscurant sources were placed on a vertical line with a spacing of 50 meters between each source. The *x-y* coordinates of these sources ranged from (0.0,-500.0) to (0.0,500.0), thus the 11[th] source is located at the origin. One observer was placed at (-1000.0,0.0) with the targets located along another vertical line at (2000.0,0.0). These target locations ranged from *x-y* coordinates (2000.0,-550.0) to (2000.0,550.0), with a inter-target spacing of 100 meters. See Figure 4 for a graphical representation of this setup and Table 1 for the meteorological parameters of the test.

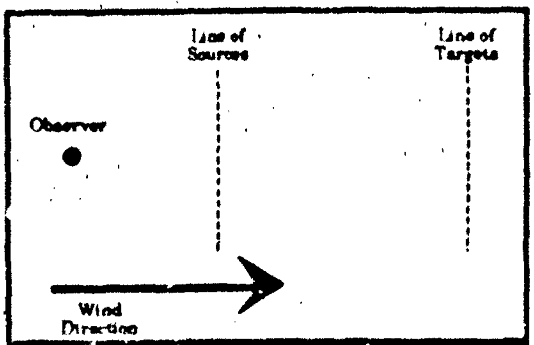

Figure 4. Scenario Setup

Table 1: Meteorological Parameters

Relative Humidity - 50%
Wind Speed - 5 m/s
Pasquill Category - 4
Air Temperature - 25°C
Air Pressure - 1013 mb
Site Latitude - 32° N
Wind Direction - 270 degrees

Results

As shown in Table 2, agreement between COMBIC and the new VIC representation was surprisingly close for the Hexachloroethane Smoke Pot and for the Red Phosphorus Grenades (10-20 and 4-6 percent error respectively). The 155 White Phosphorous (WP) comparison was another matter, yielding a 30-80 percent error in results. Closer investigation suggests the source of this error is probably due to the large amount of latent heat within the WP cloud. This heat causes the cloud to rise faster - a factor which the new smoke representation does not include.

Table 2: Transmission Over Time

Time (sec)	Hexachloroethane Pot		155 White Phosporous		Grenades	
	New	COMBIC	New	COMBIC	New	COMBIC
50	0.840	0.937	0.033	0.156	0.853	0.850
100	0.817	0.862	0.155	0.371	0.867	0.816
150	0.827	0.821	0.279	0.529	0.890	0.823
200	0.822	0.768	0.388	0.605	0.906	0.844
250	0.816	0.721	0.447	0.642	0.918	0.861
300	0.829	0.698	0.509	0.706	0.930	0.879

Spectral Band (3.0 - 5.0)

COMBIC transmissions are considering contributions from plumes ONLY

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Conclusions

The new methodology outlined in this paper provides the VIC model with an entirely new capability - representing the effect of obscurants on the realistic battlefield. It has all the aspects of a good low resolution model, namely, that it yields a close approximation to the reality-model; provides those answers relatively fast, and is easy to understand. There is still more work to be done on this model however, as its inclusion into the VIC model has resulted in a substantial increase in runtime - as much as two or three times the runtime was required in the CSS scenarios. Still, it is a simplistic approach that yields realistic results to a complex problem.

Appendix A

Derivation of the Screen Volume Equation

In order to determine the volume of a smoke/dust screen, its representation must be analyzed and the main features determined. (See Figure 1) These screen *descriptors* are defined below.

S	The length of the source baseline
B	The distance across the downwind edge of the screen
X_{ld}	The distance from the sources to the downwind edge
Y_{ld}	The crosswind drift
Z_{ld}	The height attained

The variables X_{ld}, Y_{ld}, and Z_{ld} are obtained by accessing the COMBIC generated downwind history profile for the source type producing the screen. The other variables are generated by the VIC model and are dependent on a number of factors, such as the deploying unit radius, the type of mission, etc.

An equation for the screen volume can now be derived from the variables listed above. Referring to Figure A-1, the reader can see that the screen volume can be obtained by summing the volume of several smaller solids - a rectangular solid in the center and the two triangular wedges on either side.

$$V_s = V_r + 2V_t \qquad (A\text{-}1)$$

where

V_r	the volume of the rectangular region
V_t	the volume of a triangular wedge, and
V_s	the volume of the screen

The volume of the rectangular solid is easily obtained. This region is actually a wedge; so its volume is simply given by

$$V_r = \frac{S X_{ld} Z_{ld}}{2} \qquad (A\text{-}2)$$

Arriving at an equation for V_t, on the other hand, is a bit more difficult. The triangular wedge is essentially a pyramid with a rectangular base (See Figure 2). Due to the unusual appearance of this solid, integral calculus was used to determine its volume. By choosing to integrate along the x-axis, each integral slice yields a rectangular cross section. The boundaries of these cross-sectional areas lie along a line going from (0,0) to (X_{ld}, Y_{ld}) in the y-direction, and from (0,0) to (X_{ld}, Z_{ld}) in the z-direction. The volume of an integral slice can then be found by the formula

$$V_t = yz\,dx \qquad (A\text{-}3)$$

where

V_t	Volume of integral slice
dx	depth of cross-section
y	width of cross-section
z	height of cross-section

To be able to integrate this volume, the terms y and z must be rewritten in terms of x. Thus we have

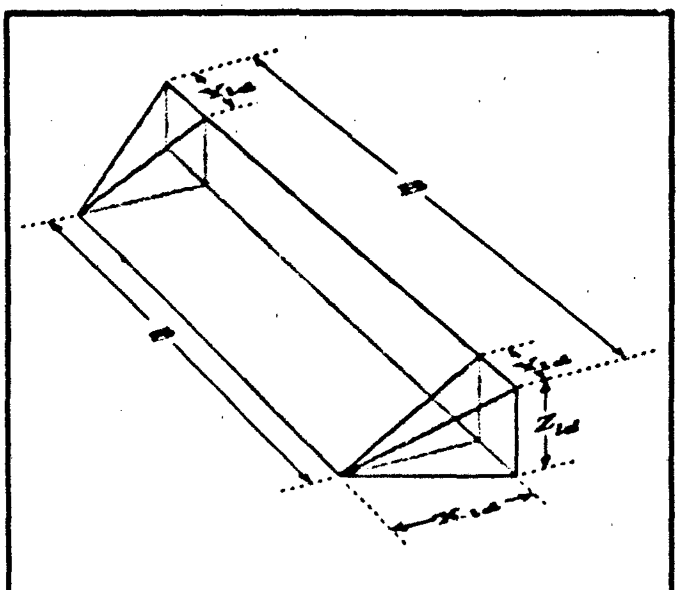

Figure A-1. COMBIC Smoke Screen Descriptors

$$y = \left(\frac{Y_{ld}}{X_{ld}}\right)x \text{ and } z = \left(\frac{Z_{ld}}{X_{ld}}\right)x \qquad (A\text{-}4)$$

Now substituting into the equation for V_t yields

$$V_t = yz\,dx = \left(\frac{Y_{ld}}{X_{ld}}\right)x\left(\frac{Z_{ld}}{X_{ld}}\right)x\ dx, \text{ or}$$

$$V_t = \left(\frac{Y_{ld}Z_{ld}}{X_{ld}^2}\right)x^2\ dx \qquad (A\text{-}5)$$

Now integrating from 0 to X_{ld} gives

$$V_t = \int_0^{X_{ld}} \frac{Y_{ld}Z_{ld}}{X_{ld}^2}x^2\ dx = \frac{X_{ld}Y_{ld}Z_{ld}}{3} \qquad (A\text{-}6)$$

Substituting the results of equations A-2 and A-6 into equation A-1 yields

$$V_e = V_r + 2V_t$$

$$= \frac{S X_{ld} Z_{ld}}{2} + 2\frac{X_{ld}Y_{ld}Z_{ld}}{3}$$

$$= X_{ld}Z_{ld}\frac{3S + 4Y_{ld}}{6}$$

$$= \left(\frac{X_{ld}Z_{ld}}{6}\right)(S + 2(S + 2\ Y_{ld}))$$

and recalling from Figure A-1 that

$$S + 2\,Y_{ld} = B$$

yields the desired result

$$V_e = \left(\frac{X_{ld}Z_{ld}}{6}\right)(S + 2B) \qquad (A\text{-}7)$$

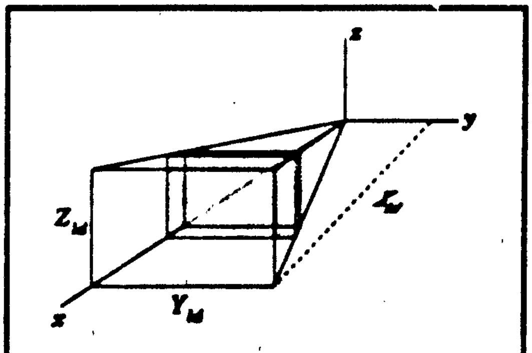

Figure 2. Determination of the Volume of a Triangular Wedge

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[2] Hoock, D. W., et al, *Combined Obscuration Manual for Battlefield Induced Contaminants - COMBIC*, US Army Atmospheric Sciences Laboratory, White Sands Missile Range, NM.

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